Interpolation

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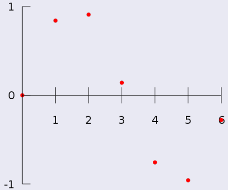
Definitions

1. Interpolation: method of constructing new data points from sampling or experimentation.

Case study

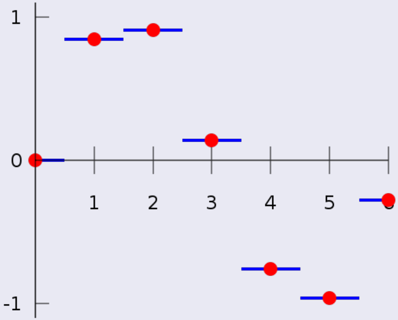
Spoze we have

x	f(x)
0	0
1	0.8415
2	0.9093
3	0.1411
4	-0.7568
5	-0.9589
6	-0.2794



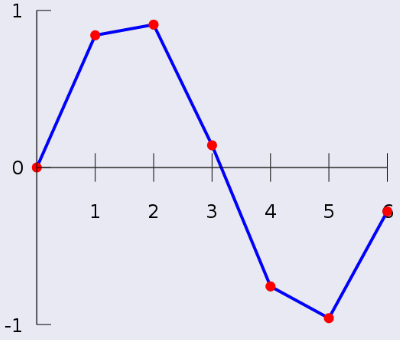
Case study

Piecewise constant



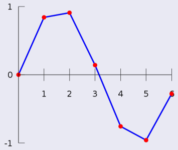
Case study

Piecewise linear



Case study

Piecewise linear



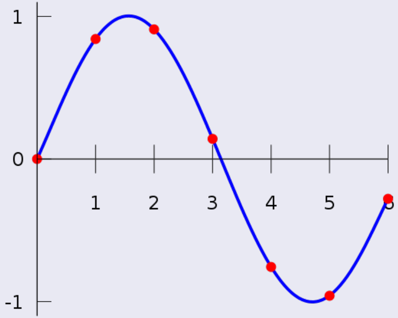
Given two data points, say (x_a, y_a) and (x_b, y_b), the interpolant function is given by:

$$y = y_a + (y_b - y_a)\frac{x - x_a}{x_b - x_a}$$

Case study

Polynomial

$$f(x) = -0.0001521x^6 - 0.003130x^5 + 0.07321x^4 - 0.3577x^3 + 0.2255x^2 + 0.9038x$$



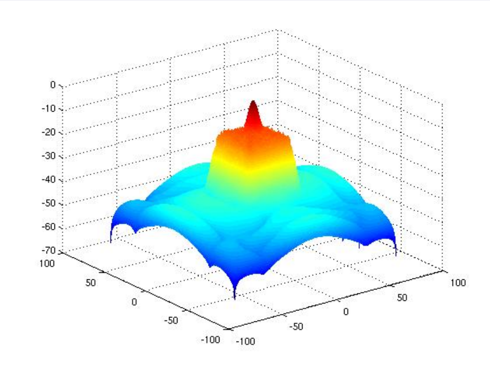
Other methods

Interpolation Methods

- Piecewise polynomial
- Spline
- Barycentric coordinates for interpolating on a triangle or tetrahedron
- Gaussian process
- And others...

2D functions

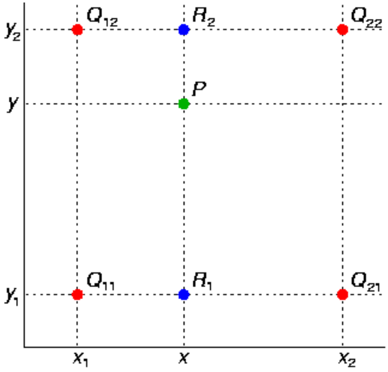
Extend concept to multivariate functions



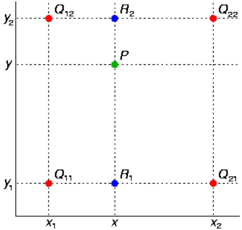
Bilinear interpolation

Problem setting

Suppose you have a function $f(x, y)$. You know the value of that function for a limited number of points (e.g., 4 points). Your goal is to approximate the function at arbitrary points (x, y).



Bilinear interpolation

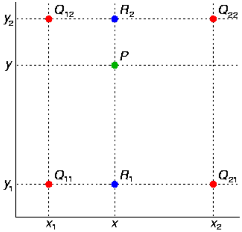


We first do linear interpolation along the X-axis:

$$R_1 = f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

$$R_2 = f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

Bilinear interpolation



We then do linear interpolation along the Y-axis:

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2)$$

$$f(x, y) \approx \frac{1}{(x_2 - x_1)(y_2 - y_1)} (f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1))$$