Symbolic Mathematics

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Overview

Symbolic

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So far in this course we dealt with MATLAB variables that were placeholders for numeric types (e.g., scalars, vectors, matrices), with one exception, anonymous functions: `f = @(x) ...`

We will now introduce the **symbolic** MATLAB data type. This is a non-numeric data type, used by the MATLAB Symbolic Math Toolbox to solve equations analytically, integrate and differentiate.

Symbolic Math

Symbolic Variables

To create three symbolic variables x, y and z, the following syntax is used:

```
>> syms x y z
```

Notice the lack of commas.

```
>> whos
  Name        Size              Bytes  Class     Attributes
  x           1x1                 112  sym
  y           1x1                 112  sym
  z           1x1                 112  sym
```

Symbolic Expressions

Symbolic expressions are created using symbolic variables. For example:

```
>> syms x y z
>> f = x.^2 + y - z


f =


x^2 + y - z
```

It can also be created using the `sym` function:

```
f = sym('x.^2 + y - z')
```

Utilities

Substitution

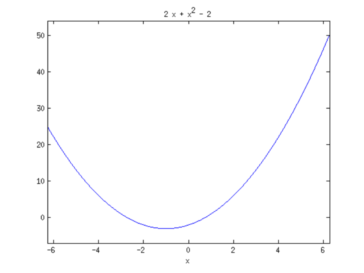
Symbolic expressions can be changed. One useful operation is substitution. The MATLAB function `subs` does that. The syntax is as follows:
`subs(S, old, new)`.
For example:

```
>> f = sym('x^2 + y - z');
>> subs(f, 'x', 'a')


ans =


a^2 + y - z
```

Plotting

MATLAB symbolic toolbox provides a function to plot symbolic expressions of one variable: `ezplot(S)`, where `S` is the symbolic expression. Example:

```
>> f = sym('x^2 + 2*x - 2');
>> ezplot(f)
```



Utilities

Expansion

MATLAB symbolic toolbox provides functions to manipulate algebraic expressions. For example `expand(S)`:

```
>> f = sym('(x + 2) * (x + 1)');
>> expand(f)

ans =

x^2 + 3*x + 2

```

performs an expansion of `f`.

Utilities

Factorization

`factor(S)`:

```
>> f = sym('x^2 + 3*x + 2');
factor(f)


ans =


(x + 2)*(x + 1)
```

performs the factorization of `f`.

Utilities

Simplification

`factor(S)`:

```
>> syms  x a b c
>> simplify(exp(c*log(sqrt(a+b))))

ans =

(a + b)^(c/2)
```

performs the simplification of `f`.

Utilities

Pretty

factor(S):

```
>> syms  x a b c
>> S = simplify(exp(c*log(sqrt(a+b))))
S =

(a + b)^(c/2)

>> pretty(S)
ans =
        c/2
(a + b)
>> S = sym('2*x^2 + 3*x - 2');
>> pretty(S)
   2
2 x  + 3 x - 2
```

Equation Solving

The `solve` function is used to solve equations. For example:

```
 >> S = sym('x^2 + 2 = 0');
>> solve(S)

ans =

  i
 -i
```

Two complex solutions.

Equation Solving

```
>> S = sym('sin(x) = 2*pi');
>> solve(S)

ans =

      asin(2*pi)
 pi - asin(2*pi)
```

Infinite number of solutions, since $a \in \mathcal{R}$.

Utilities

Differentiation

The `diff` function performs analytic differentiation.

```
>> S = sym('sin(x)');
>> diff(S)

ans =

cos(x)
```

Utilities

Differentiation

Another example:

```
>> S = sym('sin(x) + cos(x) - 2*x^2 + 2');
>> diff(S)

ans =

cos(x) - 4*x - sin(x)
```

Utilities

Integration

The `int(S)` function returns the indefinite integral of a symbolic expression `S`.

```
>> S = sym('cos(x)');
>> int(S)

ans =

sin(x)
```

$\int cos(x) = sin(x)$

Utilities

Integration

The `int(S, 1, 2)` function returns the definite integral of a symbolic expression `S`, evaluated in the range $[1, 2]$.

```
>> S = sym('cos(x)');
>> int(S)

ans =

sin(x)
```

$\int cos(x, 1, 2)|_1^2 = sin(2) - sin(1)$