Homework Solutions – Chapters 3-5

Homework 3.1

1. The approximate probability is 103/186 = 0.55 or about 55%.
2. 43% = 0.42953 = (23+55+33+17)/(48+122+23+55+33+17)
3. 6*5*3=90
4. Choose the 4 who will go on the search on the 4 paths:

$$\binom{9}{4} = \frac{9!}{4!\,(9-4)!} = \frac{9*8*7*6*5!}{4!*5!} = \frac{9*8*7*6}{4*3*2} = 126$$

   Or choose the 5 who will stay:

$$\binom{9}{5} = \frac{9!}{5!\,(9-5)!} = \frac{9*8*7*6*5!}{5!*4!} = \frac{9*8*7*6}{4*3*2} = 126$$

   Either approach is the same!

5. $P_{9,4} = \frac{9!}{(9-4)!} = \frac{9*8*7*6*5!}{5!} = 9*8*7*6 = 3024$

6. First digit has 8 possibilities (2,3,4,5,6,7,8,9). All other digits have 10

   Answer=8*10*10*10*10*10*10=8,000,000

7. $\binom{52}{13} = 635{,}013{,}559{,}600$. About 635 Billion!
8. A. Num ways to pick 2 beers: $\binom{5}{2} = 10$

   B. Num ways to pick to cans: $\binom{15}{2} = 105$

   C. Prob(pick 2 beers) = $\frac{\binom{5}{2}}{\binom{15}{2}} = \frac{10}{105} = 0.095$

Homework 3.2

1.
   a. 0.95 (=0.9+0.2-0.15)
   b. 0.75 (=0.9-0.15)
   c. 0.05 (=0.2-0.15)
   d. 0.05 (=1-0.95)
   e. 0.75 (same as *b*)
2.
   a. 0.15 (=30/200)
   b. 0.1 (=20/200)

c. 0.3 (=1-140/200)
d. 0.05 (=0.3-0.15-0.1)
e. 0.2 (=0.15+0.05)
f. 0.15 (=0.1+0.05)

A bit more detail for the answers above:

```
given in problem statement:

   (1) P(neither)=P( not(C u D) ) = 140/200 = 0.7
   (2) P(C intersect not D) = 20/200 = 0.1 (this is NOT the whole circle)
   (3) P(not C intersect D) = 30/200 = 0.15 (this is NOT the whole circle)

(a) P(only a dog) = P(not C intersect D) = (given) = 0.15
(b) P(only a cat) = P(C intersect not D) = (given) = 0.1
(c) P(either) = P(C u D) = 1 - P( not(C u D) ) = 1 - 0.7 = 0.3
        or = 1 - P(neither) = 1 - 0.7 = 0.3
(d) P(dog and cat) = P(D intersect C) = two ways to solve
(i) draw picture, subtract a and b from c
(ii) do e and f, then use addition rule.
(e) P(dog) = 0.15 + 0.05 = 0.2 (the two parts of the Dog circle)
(f) P(cat) = 0.1 + 0.05 = 0.15 (the two parts of Cat circle)
```

3.

a. 0.2 (=0.7+0.4-0.9)
b. 0.1 (=1-0.9)
c. 0.2 (=0.4-0.2)
d. 0.5 (=0.7-0.2)
e. 0.7 (=0.5+0.2)

4.

a. 0.80 (=1-0.20)
b. 0.30 (=110-80)
c. 0.10 (=0.4-0.30)
d. 0.60 (=1-0.40)

5.

a. 15% (this was given in the problem statement)
b. 17% (=15%+7%-5%)
c. 2% (=7%-5%)
d. 83% (=100%-17%)
e. 12% (=10%+2%)

6. Let
   A = faces the same
   B = sum of the faces is 9 or more
   $P(A \cap B) = P(\ (5,5)\ and\ (6,6)\ ) = 2/36$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{10}{36} - \frac{2}{36} = \frac{14}{36} = 0.389$$

7. Let
   A = Heart
   B = King
   $P(A \cap B) = P(\ King\ of\ Hearts\ ) = 1/52$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = 0.308$$

Homework 3.3

1.
   a. 0.167 (=0.15/0.9)
   b. 0.75 (=0.15/0.2)
   c. No, they are dependent because P(G|S)=0.75 does not equal P(G)=0.9
2.
   a. 0.5 (=0.1/0.2)
   b. Yes, they are independent because P(H|G)=0.5 equals P(H)=0.5

3.
   a. 0.75 (=P(M|S)=0.3 / 0.4)
   b. 0.43 (=P(S|M)=0.3 / 0.7)
   c. 0.57 (=P(not S | M)=0.4/0.7) or P(not S | M) = 1-P(S | M) = 1 – 0.3/0.7)

4. Check: P(M) = P(M|S):

   P(M) = 0.15
   P(M|S) = 0.05/0.07 = 0.71, Thus, not independent.

   Or, check: P(S) = P(S|M)

   P(S) = 0.07
   P(S|M) = 0.05/0.15 = .33, Thus, not independent.

5. $P(H|K) = \frac{P(H \cap K)}{P(K)} = \frac{P(K_H)}{P(K)} = \frac{1/52}{4/52} = 1/4$

6. $P(K|H) = \frac{P(H \cap K)}{P(H)} = \frac{P(K_H)}{P(H)} = \frac{1/52}{13/52} = 1/13$

7. $P(DietCoke|Soda) = \frac{P(DietCoke \cap Soda)}{P(Soda)} = \frac{P(DietCoke)}{P(H)} = \frac{10/25}{15/25} = \frac{10}{25} = 0.4$

8.
   a. 1/6 * 1/6 * 1/6 = 1/216 = 0.00463
   b. 1/6 * 1/6 * 1/6 = 1/216 = 0.00463
   c. 4/6 * 5/6 * 3/6 = 60/216 = 0.278

Homework 3.4

1.

a. (420+140)/1400 = 560/1400 = 0.4
b. 350/1400 = 0.25
c. 490/1400 = 0.35
d. (140+350)/1400 = 490/1400 = 0.35
e. (140+350+490)/1400 = 980/1400 = 0.7
more detail:

```
two ways to do it:

(1)

P(F)=490/1400=0.35
P(G)=840/1400=0.60
P(F intersect G) = 350/1400 = 0.25

P(F u G) = 0.35 + 0.6 - 0.25 = 0.70

(2)

Out of the 4 cells, count the 3 cells that are "Obama or female"

(140+490+350)/1400 = 0.7
```

f. 0.714 (=0.25/0.35)
g. P(M&B)=420/1400=0.3, P(B)=(420+140)/1400=0.4. Thus, P(M|B) = 0.3/0.4 = 0.75 (answer)
h. No, they are dependent because P(M|B)=0.75 does not equal P(M) = 910/1400 = 0.65

2.

a.

		Car		
		Y	N	Tot
Phone	Y	42	28	70
	N	126	11	137
	Tot	168	39	207

		Car		
		Y	N	Tot
Phone	Y	20%	14%	34%
	N	61%	5%	66%
	Tot	81%	19%	

b. 34%
c. 81%
d. 66%
e. 20%
f. 14%
g. 5%
h. 25% (=0.2/0.81)
i. 59% (=0.2/0.34)
j. 74% (=0.14/0.19)
k. 26% (=0.05/0.19)
l. No, they are dependent because P(P|C)=25% does not equal P(P)=34%.

Homework 4.1

1. 0.054 (=3/55)
2. 0.491 (=27/55)
3. 0.654 (=36/55)
4. 0.891 (=49/55)
5. 0.054 (=3/55)
6. 0.218 (=12/55)
7. 2/12=1/6
8. 6/12=1/2
9. 6/12=1/2
10. 4/12=1/3
11. c=14
12. c=1/36

Homework 4.2

1. 3.4
2. $0.663 = \sqrt{12 - 3.4^2}$
3. $\mu = E[X] = 4.667$
4. $V[X] = 24 - 4.667^2 = 2.219$
5. $\sigma = \sqrt{V[X]} = \sqrt{24 - 4.667^2} = 1.490$
6. $\mu = E[X] = -1.2$
7. No, because the expected return is negative. This means that on average, if you play this game many times, you will *lose* $1.20 each time you play.
8.

   a. $\mu = -1.2$ (from a previous example)

   b. $E[X^2] = \sum x^2 f(x) = (5)^2(0.4) + (2)^2(0.2) + (-9)^2(0.4) = 43.2$

   c. $\sigma^2 = V[X] = E[X^2] - \mu^2 = 43.2 - (-1.2)^2 = 41.76$

   d. $\sigma = \sqrt{V[X]} = \sqrt{41.76} = 6.46$

Homework 4.3

1. (a) n=6, p=0.05 (b) $P(X = 2) = binompdf(6,0.05,2) = 0.0305$
2. (a) n=20, p=0.95 (b) $P(X \geq 17) = 1 - P(X \leq 16) = 1 - binomcdf(20,0.95,16) = 0.9841$
3. $\mu = E[X] = 250(0.05) = 12.5$
4. (a) n=60, p=0.15+0.15=0.3 (b) $P(X > 20) = 1 - P(X \leq 20) = 1 - binomcdf(60,0.3,20) = 0.2378$
5.

(a) n=4, p=0.375, $P(X = 4)$

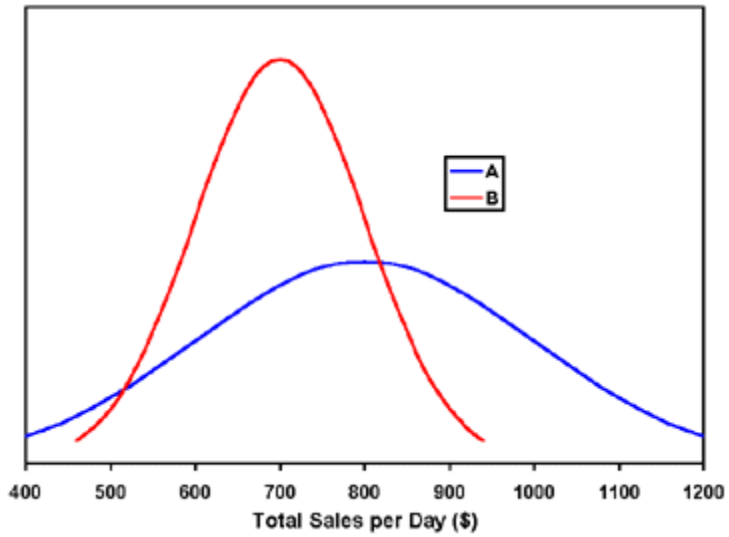
(b) $P(X = 4) = binompdf(4,0.375,4) = 0.0198$

(c) $P(X = 0) = binompdf(4,0.375,0) = 0.1526$

(d) $\mu = E[X] = 162(0.1526) = 24.7$

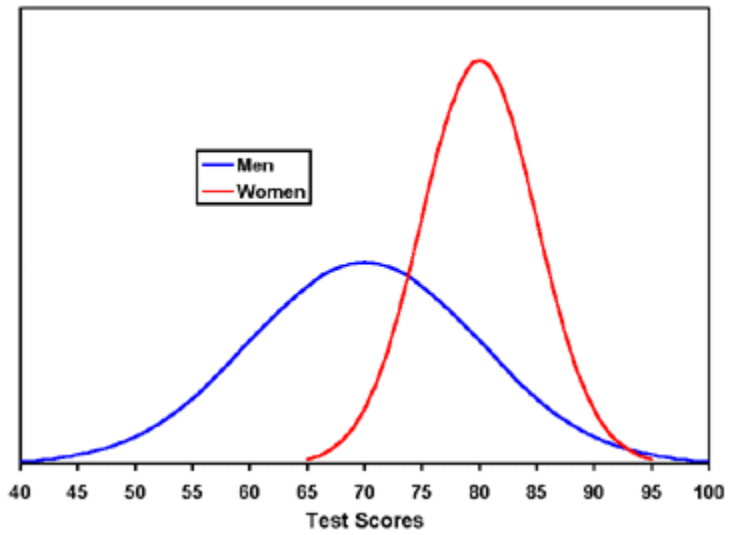
Homework 5.1

1. (a)



(b) Restaurant A is more likely to make less than $500 in a day because the area to the left of 500 on the blue curve (A) is *larger* than the area to the left of 500 on the red curve (B).

2. (a)



(b) The area to the right of 90 is *larger* for men than for women. This is hard to see, but it is true.

Homework 5.2

1.

(a) $P(A < 500) = normalcdf(-1EE99, 500, 800, 200) = 0.0668$

$P(B < 500) = normalcdf(-1EE99, 500, 700, 100) = 0.0228$

Thus, Restaurant A is more likely to have sales below 500

(b) $P(600 < A < 1000) = normalcdf(600, 1000, 800, 200) = 0.6827$

(c) $P(B > 865) = 1 - P(B < 865) = 1 - normalcdf(-1EE99, 865, 700, 100) = 0.0495$

2.

(a) $P(65 < X < 73) = normalcdf(65, 73, 76, 5) = 0.2603$

(b) $P(X < 65 \text{ or } X > 73) = 1 - P(65 < X < 73) = 1 - 0.2603 = 0.7397 = 73.97\%$

(c) $P(X = 90) = 0$

(d) $P(X < 70) = normalcdf(-1EE99, 70, 76, 5) = 0.1151$

3.

(a) $P(X < 1200) = normalcdf(-1EE99, 1200, 1500, 200) = 0.0668$

(b) $P(1200 < X < 1700) = normalcdf(1200, 1700, 1500, 200) = 0.7745$

(c) $P(X > 1700) = 1 - P(X < 1700) = 1 - normalcdf(-1EE99, 1700, 1500, 200) = 0.1586$

(d) $P(X < 1200 \text{ or } X > 1700) = 1 - P(1200 < X < 1700) = 1 - 0.7745 = 0.2255$

Homework 5.3

1.

(a) $invNorm(0.15, 70, 15) = 54.45$

(b) $invNorm(0.80, 80, 5) = 84.21$

2.

(a) $invNorm(0.93, 1500, 200) = 1795.2 \approx 1795$

(b) $invNorm(0.25, 1500, 200) = 1365.1$

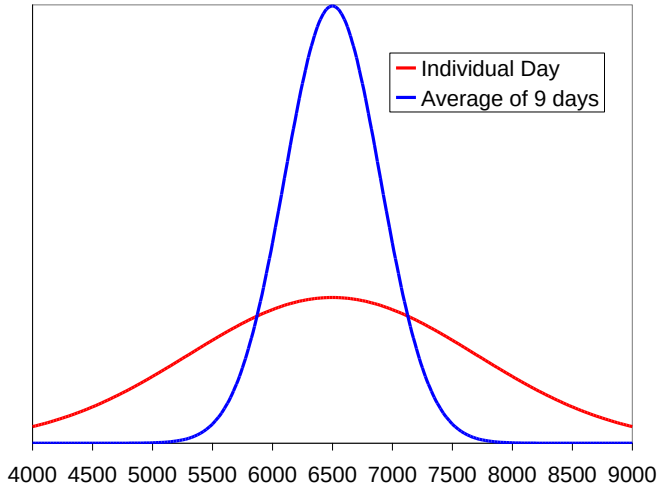
Homework 5.4

1.

(a) $P(\overline{X} < 6000) = normalcdf\,(-1EE99, 6000, 6500, \frac{1200}{\sqrt{9}}) = 0.1056$

(b) $\sigma_{\bar{x}} = \frac{1200}{\sqrt{9}} = 400$

(c)



(d) $P(\overline{X} < 7000) = normalcdf\,(-1EE99, 7000, 6500, \frac{1200}{\sqrt{25}}) = 0.9814$

Homework 5.5

1.

(a) $P(\overline{X} < 405) = normalcdf\,(-1EE99, 405, 400, \frac{18}{\sqrt{36}}) = 0.9522$

(b) The Central Limit Theorem

(c) $P(397 < \overline{X} < 403) = normalcdf\,(397, 403, 400, \frac{18}{\sqrt{36}}) = 0.6827$